Reading Time: An initial **2 minutes** to view **BOTH** sections

MATHEMATICS METHODS : UNITS 3 & 4, 2021

Test 4 – (10%)

4.1.9 – 4.1.14, 4.2, 4.3.1 – 4.3.3

Time Allowed	**First Name**	**Surname**	**Marks**
22 minutes	**MARKING GUIDE**		22 marks

Circle your Teacher's Name:

Mrs Alvaro	Mrs Bestall	Ms Chua
Mr Gibbon	Mrs Greenaway	Mr Luzuk
Mrs Murray	Ms Robinson	Mr Tanday

Assessment Conditions: ***(N.B. Sufficient working out must be shown to gain full marks)***

- Calculators: Not Allowed
- Formula Sheet: Provided
- Notes: Not Allowed

PART A – CALCULATOR FREE

Question 1 **[2, 2 – 4 marks]**

Differentiate the following with respect to x.

a) $f(x) = \sin(5x)\ln(x)$

$$f'(x) = \sin(5x)\frac{1}{x} + \ln(x)\cos(5x)(5)$$
$$= \frac{1}{x}\sin(5x) + 5\ln(x)\cos(5x)$$

✓ First term correct
✓ Second term correct

b) $f(x) = \ln\left[\frac{(x+2)^3}{x}\right]$

$$f(x) = \ln(x+2)^3 - \ln(x)$$
$$= 3\ln(x+2) - \ln(x)$$
$$f'(x) = 3\frac{1}{x+2} - \frac{1}{x}$$
$$= \frac{3}{x+2} - \frac{1}{x}$$
$$= \frac{3(x+2)^2}{(x+2)^3} - \frac{1}{x}$$
$$= \frac{2x-2}{x^2+2x}$$

✓ Simplifies using log laws OR uses quotient rule correctly

✓ Correct differentiation
Accept any correct version of the simplification

NB: No penalty for using quotient rule – this is a self-imposed time penalty!

Question 2 **[1, 5 – 6 marks]**

Consider the function $f(x) = x - \ln(x)$.

a) Find the values for which $f(x)$ is defined.

$f(x)$ is defined for $x > 0$

✓ Correct domain

0 marks for "asymptote at $x = 0$"
0 marks if wrong ineaquality

a) Determine the coordinates and the nature of the stationary point of $f(x)$.

$$f'(x) = 1 - \frac{1}{x}$$
$$= \frac{x-1}{x}$$
$$0 = \frac{x-1}{x}$$
$$x = 1$$
$$f''(x) = \frac{1}{x^2}$$
$$f''(1) \Rightarrow \textit{positive}$$

There is a minimum turning point at (1,1)

✓ First derivative

✓ Sets to 0 and solves for x

✓ Checks second derivative

✓ States coordinates of SP
✓ States nature of SP

Accept $f(1) = 1 - \ln(1)$

[teaching point!]

Question 3 **[2 marks]**

Find the integral of $\frac{6x^2-8}{x^3-4x+1}$.

$$\int \frac{6x^2-8}{x^3-4x+1}dx = 2\int \frac{3x^2-4}{x^3-4x+1}dx$$
$$= 2\ln(x^3-4x+1)+c$$

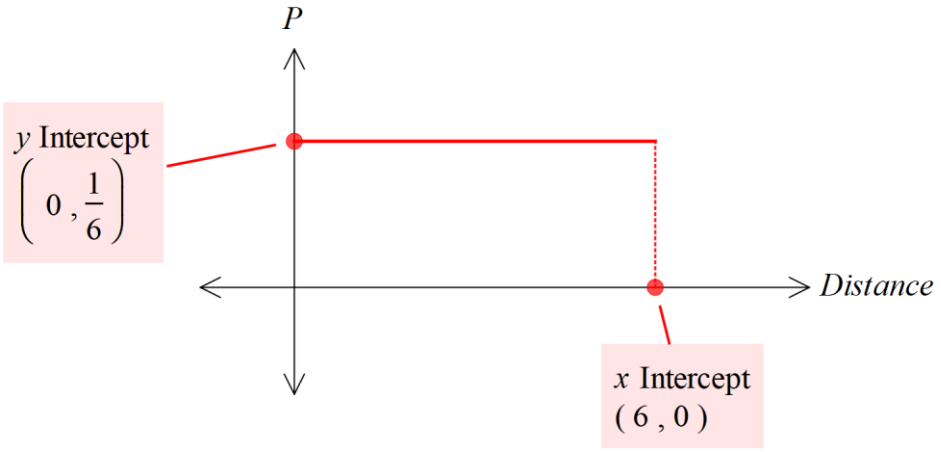
✓ Correct integral

✓ Includes constant

Question 4 **[2, 1, 1 – 4 marks]**

An unreinforced concrete path is equally likely to crack anywhere along its length. An unreinforced path is 6m long.

a) Construct a probability density graph for the distance of the first crack from the beginning of the path.



✓ Correct domain

✓ Correct *y*-intercept

No penalty for not labelling axes [teaching point]
-1 if vertical line solid
No penalty if no vertical line

b) Find the probability that the first crack is 2.3m from the beginning of the path.

$P(X = 2.3) = 0$

✓ Correct probability

c) Find the probability that the first crack is between 2.25m and 2.35m from the beginning of the path.

$P(2.25 < X < 2.35) = \frac{1}{60} \approx 0.0167$

✓ Correct probability

0 marks for a fraction that includes a decimal

Question 5 **[2 marks]**

A continuous random variable X is transformed to the random variable Y according to the equation $Y = 2X + 3$. The mean and standard deviation of X are 27.8 and 5.6 respectively. What are the mean and standard deviation of Y?

$E(Y) = 2E(X) + 3$
$= 2 \times 27.8 + 3$
$= 58.6$

$\sigma_Y = 2\sigma_X$
$= 2 \times 5.6$
$= 11.2$

✓ Correct E(X)
✓ Correct σ

Question 6 **[2, 2 – 4 marks]**

For each of the following cases, state whether the sampling method is biased and justify your reasoning.

a) A Year 11 student asks each of the people in his class what kind of mobile they have and how many SMSs they send each week to determine mobile phone use among high school students.

Yes, it is biased; only asked his year group, students may not accurately recall the number of SMSs they send each week.

✓ States biased
✓ Gives reasonable justification

b) A market research company rings 100 phone numbers taken at random from the residential phone directory to ask whether they vacationed in WA, interstate or overseas in the last 3 years as part of a study for the tourism industry of WA.

Yes, it is biased; not everyone has a listed phone number, some people may not answer.

✓ States biased
✓ Gives reasonable justification

Reading Time: An initial **2 minutes** to view **BOTH** sections

MATHEMATICS METHODS : UNITS 3 & 4, 2021

Test 4 – (10%)

4.1.9 – 4.1.14, 4.2, 4.3.1 – 4.3.3

Time Allowed	**First Name**	**Surname**	**Marks**
28 minutes	**MARKING GUIDE**		27 marks

Circle your Teacher's Name:

Mrs Alvaro Mrs Bestall Ms Chua
Mr Gibbon Mrs Greenaway Mr Luzuk
Mrs Murray Ms Robinson Mr Tanday

Assessment Conditions: ***(N.B. Sufficient working out must be shown to gain full marks)***

- Calculators: Allowed
- Formula Sheet: Provided
- Notes: Not Allowed

PART B – CALCULATOR ALLOWED

Question 7 **[1, 2, 2, 2 – 7 marks]**

A continuous random variable X has the probability density function $f(x) = ax(4-x^2)$ for $0 \le x \le 2$.

a) Find a.

$\int_0^2 ax(4-x^2)dx = 1$

$a = 0.25$

✓ Correct value

b) Find the mean of X.

$\mu = \int_0^2 0.25x^2(4-x^2)dx = \frac{16}{15}$

✓ Sets up integral
✓ Correct value (answer only ok)

c) Find the standard deviation of X.

$\sigma^2 = \int_0^2 \left(x - \frac{16}{15}\right)^2 \times 0.25x(4-x^2)dx = \frac{44}{225}$

$\sigma = \frac{2\sqrt{11}}{15} \approx 0.4422$

✓ States equation (either form)
✓ Correct value (answer only ok)

d) Find the median of X.

$\int_0^m 0.25x(4-x^2)dx = 0.5$

$m = 1.0824$

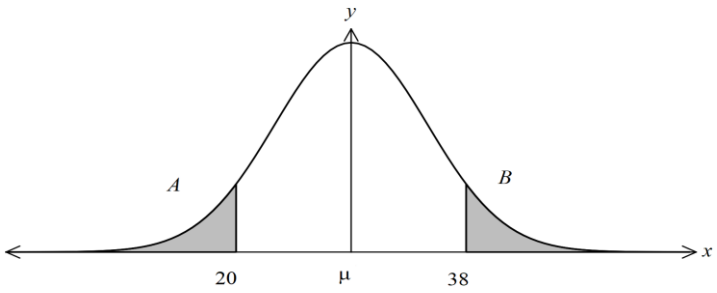
✓ States equation
✓ Correct value (answer only ok)

Question 8 **[3 marks]**

The distribution curve shown corresponds to $X \sim N(\mu, \sigma^2)$.

Area $A =$ Area $B = 0.2$.

Find μ and σ.



$P(-k < X < k) = 0.6$

$\Rightarrow k = \pm 0.84162$

$\frac{20-\mu}{\sigma} = -0.84162$ $\quad$ $\frac{38-\mu}{\sigma} = 0.84162$

Solve simultaneously: $\mu = 29$, $\sigma = 10.6937$

✓ Finds z-value(s)

✓ Sets up equations

✓ Values for μ and σ

OR

$\mu = \frac{20+38}{2} = 29$

✓ Averages upper and lower bounds to find μ

$\frac{20-29}{\sigma} = -0.84162$

$\sigma = 10.6937$

✓ Value σ

Question 9 **[1, 3, 2 – 6 marks]**

Let X be the weight in grams of bags of sugar filled by a machine. Bags less than 500g are considered underweight. Suppose $X \sim N(503, 2^2)$.

a) What percentage of bags are underweight?

$P(X < 500) = 0.066807$

$\therefore 6.68\%$

✓ Correct value

b) Find the probability that the weight of a bag differs from the mean by no more than 5g, given that it weighs less than 505g.

$P = \frac{P(498 < X < 505)}{P(X < 505)}$

✓ Correct numerator (formula or probability)

✓ Correct denominator (formula or probability)

$= \frac{0.83514}{0.84134}$

$= 0.9926$

✓ Correct value

c) If a quality inspector randomly selects 20 bags, what is the probability that 2 or fewer bags are underweight?

Let Y be the number of bags (out of 20) that are underweight.

$Y \sim Bin(20, 0.06681)$

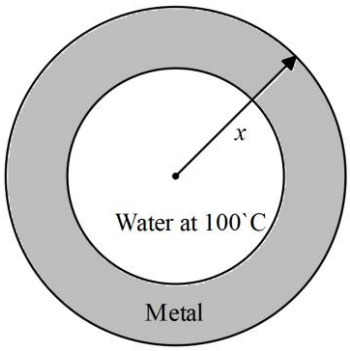
✓ States distribution and parameters

$P(Y \leq 2) = 0.8543$

✓ Correct value

Question 10 **[3 marks]**

A metal pipe has a cross-section as shown.



The outer radius is 4cm and the inner radius is 2cm. Within the pipe, the water temperature is maintained at $100°C$.

Within the metal, the temperature drops off from inside to outside according to $\frac{dT}{dx} = -\frac{10}{x}$, where x is the distance from the central axis and $2 \le x \le 4$.

Find the temperature of the outer surface of the pipe.

$$\frac{dT}{dx} = -10\left(\frac{1}{x}\right)$$

$$T = -10\int \frac{1}{x}\,dx$$

$$T(x) = -10\ln(x) + c$$

$$T(2) = -10\ln(2) + c = 100$$

$$\Rightarrow c = 10\ln(2) + 100$$

$$T(x) = -10\ln(x) + 10\ln(2) + 100$$

$$T(4) = -10\ln(4) + 10\ln(2) + 100$$

$$= 93.1°C$$

✓ Write equation for integral

OR

✓ Writes expression for $T(x)$ with constant

✓ Expression for $T(x)$

✓ Temperature for $T(4)$

Question 11 **[4 marks]**

Danielle's class scored an average of 18.8 on an English test with a standard deviation of 5.4. The same group scored an average of 22.3 on a Maths Methods test with a standard deviation of 3.6. Danielle scored 27 on both tests. In which test did she do better? Justify your reasoning.

$z_E = \frac{27-18.8}{5.4}$
$= 1.519$

$z_M = \frac{27-22.3}{3.6}$
$= 1.306$

✓ z-score for English
✓ z-score for Maths

Danielle did better in English as her score was more standard deviations above the mean.

✓ States English was the better test
✓ Justification of being further above the mean

Question 12 **[4 marks]**

Students learning to use a pottery wheel take an average time of 25 minutes to make a simple pot. 30% of such students complete their pots within 20 minutes. Assuming that the times are normally distributed, what is the probability of a student taking longer than 28 minutes?

Let X be the time taken to complete a pot.

✓ Defines random variable

$P(X < 20) = 0.3$
z-score $= -0.5244$

✓ Calculates z-score

$-0.5244 = \frac{20-25}{\sigma}$
$= 9.535$

✓ Correct value for σ

$P(X > 28) = 0.3765$

✓ Correct probability